NCPC 2016
Presentation of solutions

The Jury

2016-10-08

NCPC 2016 Jury

- Per Austrin (KTH Royal Institute of Technology)
- Pål Grønås Drange (Statoil ASA)
- Antti Laaksonen (CSES)
- Ulf Lundström (Excillum)
- Jimmy Mårdell (Spotify)
- Lukáš Poláček (Google)
- Mathias Rav (Aarhus University)
- Pehr Söderman (Kattis)
- Jon Marius Venstad (Yahoo!)

J — Jumbled Compass

Problem

Simple problem with solutions by the jury in all languages available in the contest.

Some solution (guess the language)

```
solve(N1, N2) :-
    (N2-N1 >  180 -> Ans is N2-N1-360;
     N2-N1 > -180 -> Ans is N2-N1;
                     Ans is N2-N1+360),
    write(Ans), nl.
```

Statistics: 535 submissions, 266 accepted, first after 00:03

G - Game Rank

Problem

Simulate ranking system of some vaguely familiar game.

Solution

1. Read and understand the rules.
2. Keep track of current rank, current number of stars and current number of consecutive wins.
3. Update accordingly.
4. Don't try to be clever.

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Simulate ranking system of some vaguely familiar game.

Solution

1. Read and understand the rules.
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4. Don't try to be clever.

Statistics: 960 submissions, 218 accepted, first after 00:17

D - Daydreaming Stockbroker

Problem

Play the stock market when knowing the future.

Solution (guess the language)

```
fscanf(STDIN, "%d", $days);
$money = 100;
$prev = 1<<30;
for ($i = 0; $i < $days; ++$i) {
    fscanf(STDIN, "%d", $cur);
    if ($cur > $prev)
       $money += min(floor($money/$prev), 100000)*($cur-$prev);
    $prev = $cur;
}
echo $money;
```

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       $money += min(floor($money/$prev), 100000)*($cur-$prev);
    $prev = $cur;
}
echo $money;
```

Statistics: 740 submissions, 183 accepted, first after 00:12

F - Fleecing the Raffle

Problem

When drawing p items out of $n + x$ items, what is probability that *exactly one* out of the first x items is drawn?

What is the maximum such probability over all x?

Solution

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Solution

1. Probability is
$$\frac{\binom{x}{1} \cdot \binom{n}{p-1}}{\binom{n+x}{p}} = \{\text{...some calculations...}\} = \frac{x \cdot p}{n+1} \cdot \prod_{i=2}^{x} \frac{n-p+i}{n+i}$$

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2. When going from $x - 1$ to x, probability changes by factor
$$\frac{x}{x-1} \cdot \frac{n-p+x}{n+x}$$

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2. When going from $x - 1$ to x, probability changes by factor
$$\frac{x}{x-1} \cdot \frac{n-p+x}{n+x}$$
3. Some calculus $\Rightarrow$ increase if $x < \frac{n}{p-1}$, decrease otherwise $\Rightarrow$ max happens at $x = \lfloor n/(p-1) \rfloor$.

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Solution

Linear $O(n/p)$ time solution:

```
int n, p;
scanf("%d%d", &n, &p);
int x = n/(p-1);
double res = double(x*p) / (n+1);
for (int i = 2; i <= x; ++i)
    res *= double(n-p+i) / (n+i);
printf("%.9lf\n", res);
```

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Solution

Constant time solution:

```
int n, p;
scanf("%d%d", &n, &p);
int x = n++/(p-1);
printf("%.9lf\n", x*p*exp(lgamma(n-p+x)-lgamma(n-p+1)
                         -lgamma(n+x)+lgamma(n)));
```

(But in order to do this in languages that don't provide full ISO C support, one may have to implement the Γ function oneself)

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(But in order to do this in languages that don't provide full ISO C support, one may have to implement the Γ function oneself)

Statistics: 349 submissions, 68 accepted, first after 00:17

C - Card Hand Sorting

Problem

What is minimum number of cards to move to get list of cards in some form of order?

Solution

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Problem

What is minimum number of cards to move to get list of cards in some form of order?

Solution

1. Try all $4! = 24$ possible ways of ordering the 4 suits.
2. Try all $2^4 = 16$ possible ways of choosing ascending/descending order within suits.

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Statistics: 77 submissions, 23 accepted, first after 00:29

A - Artwork

Problem

Fill horizontal and vertical blocks of squares in an initially empty grid, and output the number of unfilled connected components after each operation.

Solution

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Solution

1. The problem can be modelled as a graph where each node represents a square in the grid, and two nodes are connected if the squares belong to the same component.

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3. The process can be simulated efficiently using a union-find structure when all operations are done in the reverse order.

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Statistics: 134 submissions, 17 accepted, first after 01:14

B - Bless You Autocorrect!

Problem

Type a word using autocorrect.

Solution

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1. Realisation: may need multiple autocorrects for a single word

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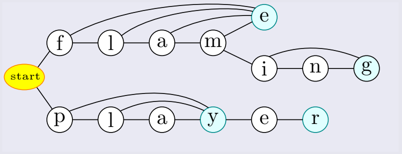
Problem

Type a word using autocorrect.

Solution

1. Realisation: may need multiple autocorrects for a single word
2. Build trie of dictionary, plus shortcut edges for autocorrects



Graph for dictionary “flame”, “flaming”, “play”, “player”

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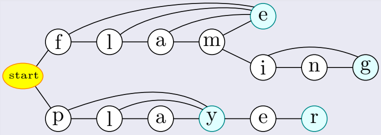

3. Find shortest distance to each node with BFS

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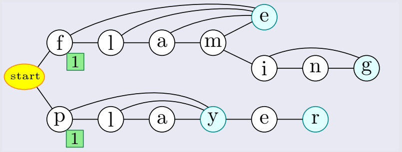

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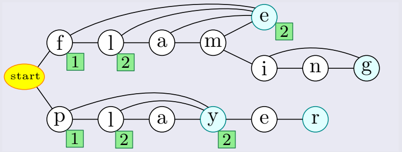

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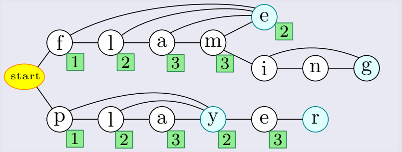

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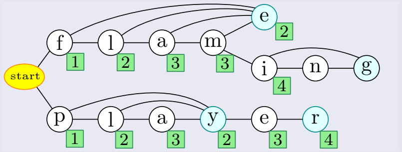

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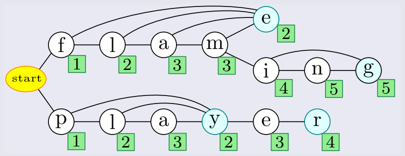

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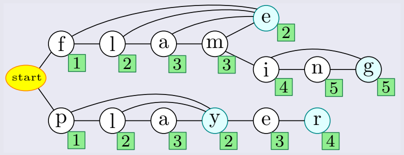

3. Find shortest distance to each node with BFS
4. To type word w, find node corresponding to longest prefix of w

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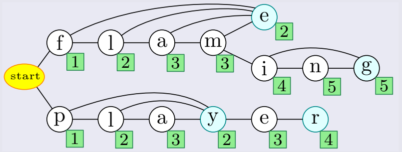

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4. To type word w, find node corresponding to longest prefix of w
5. Answer for w is distance to node + #remaining letters

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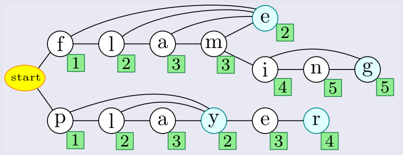

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6. Time complexity: linear in size of input.

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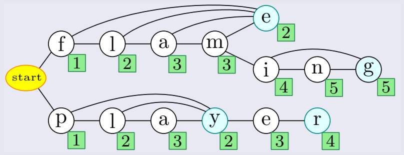

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6. Time complexity: linear in size of input.

Statistics: 70 submissions, 12 accepted, first after 00:43

K - Keeping the Dogs Apart

Problem

Two dogs move around along straight line segments, what is the closest they get to each other?

Solution 1

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Problem

Two dogs move around along straight line segments, what is the closest they get to each other?

Solution 1

1. Split the walks into intervals during which the two dogs don't switch line segments.

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Two dogs move around along straight line segments, what is the closest they get to each other?

Solution 1

1. Split the walks into intervals during which the two dogs don't switch line segments.
2. Movement is relative: the two dogs walking from P to $P + \Delta P$ and from Q to $Q + \Delta Q$ is *equivalent* to one standing still at P and other moving from Q to $Q + \Delta Q - \Delta P$

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3. Closest distance in each such interval boils down to distance between point and line segment, basic geometric primitive.

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3. Closest distance in each such interval boils down to distance between point and line segment, basic geometric primitive.
4. Time complexity: $O(n)$.

K - Keeping the Dogs Apart

Problem

Two dogs move around along straight line segments, what is the closest they get to each other?

Solution 2 (more or less the same but different perspective)

1. Split the walks into intervals during which the two dogs don't switch line segments.
2. In an interval where dogs walk from P to $P + \Delta P$ and Q to $Q + \Delta Q$, square dist. after fraction $t \in [0, 1]$ of the time is

$$\|P - Q + t(\Delta P - \Delta Q)\|_2^2 = \|P - Q\|_2^2 + 2t\langle P - Q, \Delta P - \Delta Q\rangle + t^2\|\Delta P - \Delta Q\|_2^2$$

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3. Minimum happens at $t = \frac{-\langle P - Q, \Delta P - \Delta Q\rangle}{\|\Delta P - \Delta Q\|_2^2}$ (basic calculus)
   Truncate to $t \in [0, 1]$, be careful with $\Delta P = \Delta Q$.

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$$\|P-Q+t(\Delta P-\Delta Q)\|_2^2 = \|P-Q\|_2^2+2t\langle P-Q, \Delta P-\Delta Q\rangle+t^2\|\Delta P-\Delta Q\|_2^2$$

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   Truncate to $t \in [0, 1]$, be careful with $\Delta P = \Delta Q$.
4. Time complexity: $O(n)$.

Statistics: 95 submissions, 11 accepted, first after 01:47

E - Exponial

Problem

Compute $f(n) \bmod m$, where $f(1) = 1$, $f(n) = n^{f(n-1)}$.

Solution

1. If $n \leq 5$: just compute $f(n-1)$ and then $n^{f(n-1)} \bmod m$ with modular exponentiation.
2. If $n > 5$: ??????????

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Lemma

For all n and m, and $e \geq log_2(m)$ it holds that

$$n^e \bmod m = n^{\phi(m)+e \bmod \phi(m)} \bmod m.$$

($\phi(m)$ = Euler's totient function.)

Proof: ugly and does not fit on slide. (Boils down to Chinese Remainder Theorem and ϕ being multiplicative.)

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2. If $n > 5$: compute $z = f(n-1) \bmod \phi(m)$ recursively. The lemma then says that $f(n) \bmod m = n^{\phi(m)+z} \bmod m$

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3. Time complexity:
    - each recursive call dominated by time to compute $\phi(m)$: $O(\sqrt{m})$ using naive factorization.
    - recursing until n becomes ≤ 5 hopelessly slow...

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     Lemma: $\phi(\phi(\cdots\phi(m)))$ reaches 1 after $O(\log m)$ iterations
     Proof: cute but does not fit on slide.

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     Lemma: $\phi(\phi(\cdots\phi(m)))$ reaches 1 after $O(\log m)$ iterations
     Proof: cute but does not fit on slide.

Statistics: 47 submissions, 3 accepted, first after 00:45

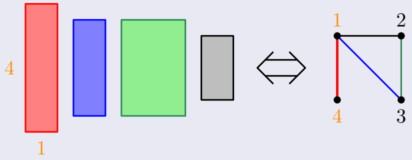
H - Highest Tower

Problem

Given a set of rectangles, build a tower of maximum height.

Solution

1. Make a graph: vertices = lengths, edges = given rectangles.



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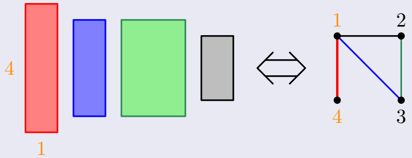
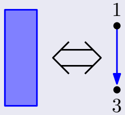
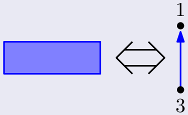
1. Make a graph: vertices = lengths, edges = given rectangles.



2. Encode orientation of a rectangle as direction of an edge.





H - Highest Tower

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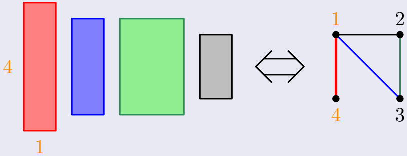

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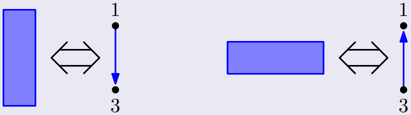

3. Orientation of rectangles gives a valid tower if no width occurs more than once $\Leftrightarrow$ all nodes have outdegree ≤ 1

H - Highest Tower

Problem Reformulated

Given an undirected graph, direct edges so that each node has at most one out-going edge and maximize $\sum_{v \in V} \text{value}(v) \cdot \text{indeg}(v)$

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Solution

1. In connected component with v vertices and e edges, average out-degree is e/v, so we must have $e \leq v$
   Each component is a tree, or a tree plus one edge.

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Statistics: 38 submissions, 3 accepted, first after 02:22

I - Interception

Problem

Given large set of paths in large graph with very special structure, find minimum set of edges that hit all paths.

Solution 1

I - Interception

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1. Try all 4 possible ways of using the two crossings.

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3. **Case 2, use one crossing**: a bit of work, better to skip and then revisit with ideas from the harder Case 3.
4. **Case 3, don't use crossings**: main challenge to handle.
   - In order to improve on Case 1, can use at most 1 extra device for the sides.
   - One side must use minimum number of crossings.

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Solution 1

Guess first position P to use after first crossing, $O(n)$ choices.

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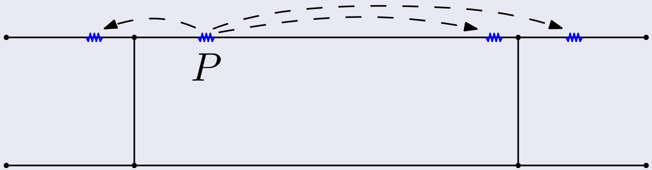

Try to squeeze the rest as close as possible to the crossing, $O(1)$ choices given P.

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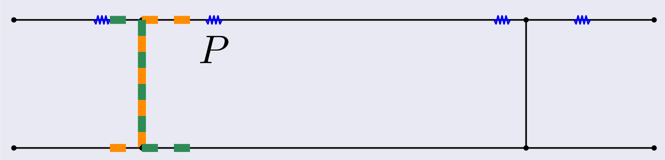

When one side is decided, the positions next to the crossings determine which crossing calls remain uncovered.

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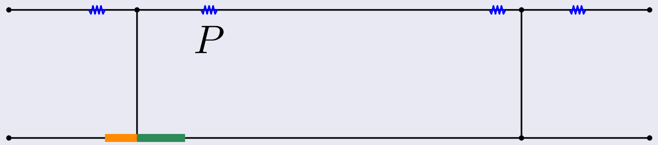

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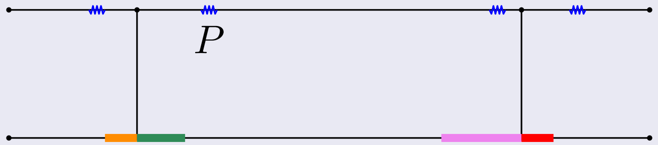

Now want optimal solution to the other side with up to 4 extra intervals added – there are $O(1)$ choices to try for the key positions.

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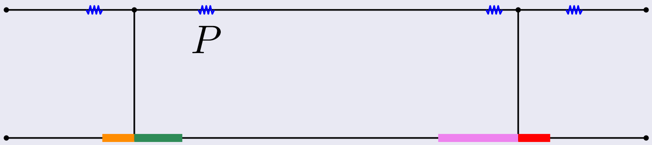

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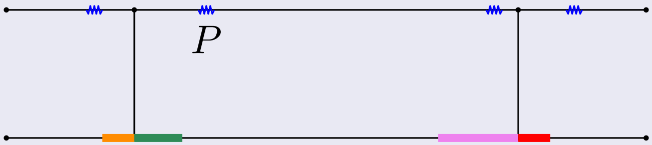

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Time complexity: $O((n+m)\log n)$ (though can be made linear at cost of making implementation even more horrible)

I - Interception

If you prefer to remain sane and don't want to take the mildly masochistic path of solution 1...

Solution 2

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1. Cover all paths contained in one of the four "tails" optimally, get first device as close to beginning as possible. *Greedy.*

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1. Cover all paths contained in one of the four "tails" optimally, get first device as close to beginning as possible. *Greedy.*
2. Add one extra device to some tails to cut off all paths from the tail to the rest of the graph. *Only* 16 *possibilities; try them all.*

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Statistics: 7 submissions, 0 accepted, first after N/A

Random numbers

296 teams with 722 contestants.

3045 submissions, 802 accepted (26%)[1]

34 number of seconds before end that last accepted submission was submitted.

482 number of lines of code used in total by the shortest **jury** solutions to solve the entire problem set. (154 of those lines for I **Interception**)

[1] These numbers only count submissions up to the first accepted solution on each problem for each team.

Random facts

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- I (**Interception**) had the largest number of test cases (125), largest amount of test data ($\approx$ 325 MB), and largest number of intentionally incorrect judge solutions (41).
- The jury wrote Python solutions for almost all problems. Exceptions: C (**Card Hand Sorting**) for no good reason, and I (**Interception**) because painful.

What now?

- Northwestern Europe Contest: November 18 in Bath (UK). Teams from Nordic, Benelux, Germany, UK, Ireland.

- Each university sends up to three(?) teams to fight for spot in World Finals (May, in Rapid City, South Dakota, USA)